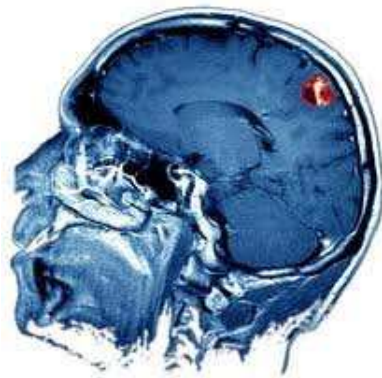


ANALYTICAL RECONSTRUCTIONS OF REGIONS OF INTEREST IN MEDICAL IMAGING

Second year two-months internship (16/06-17/09)
Made within the team GMCAO
Laboratory Pavillon Taillefer (CHU Michallon in Grenoble).



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1 Introduction

1.1 Workspace

I did my internship within the TIMC-IMAG Laboratory (Techniques for biomedical engineering and complexity management – informatics, mathematics and applications – Grenoble). The TIMC-IMAG is a collaboration of scientists and clinicians using computer science and applied mathematics for understanding and controlling normal and pathological processes in biology and health-care. This multi-disciplinary activity contributes both to the basic knowledge of those domains and to the development of systems for computer-assisted diagnosis and therapy.

1.2 Context

I worked in the GMCAO team (Biomechanical modelling, image processing, data fusion and robotics for computer-assisted medical interventions) and more specifically in the field of image processing. I was supervised by Laurent Desbats, researcher in medical imaging. He takes part to a project with the company “Surgivisio”, which is focused on research and development of advanced solutions for surgeons, using 3D imaging and surgical navigation technologies. It empowers the surgeons with innovative and efficient tools, reducing surgical time, reducing x-ray exposure, and increasing safety and accuracy of complex interventions.

1.3 Topic

I was interested by the reconstruction part of this project, and in particular the point was to reduce the x-ray exposure by decreasing the trajectory of the scanner around the patient. But this point implies a lot of modifications in the classical reconstruction way.

To start with, I had to understand principles of tomography and the mathematic theory which hides behind them. I spent a lot of time learning different ways of reconstruction. There are two schools of thought : reconstruction by analytical methods and by algebraic methods. I considered both, but since another student developed algorithms based on algebraic methods, I decided then to focus mine on analytical methods.

I read several research papers relating the state of the art of medical imaging research in the last few years. We decided to develop a Matlab program which implements reconstruction of a specific region (called region of interest) when the scanner moves on a short trajectory.

2 Definition and motivation

2.1 Principles of Tomography

Tomography refers to the cross-sectional imaging of an object from either transmission or reflection data collected by illuminating the object from many different directions. The impact of this technique in diagnostic medicine has been revolutionary, since it has enabled doctors to view internal organs with unprecedented precision and safety to the patient. The first medical application utilized x-rays for forming images of tissues based on their x-ray attenuation coefficient.

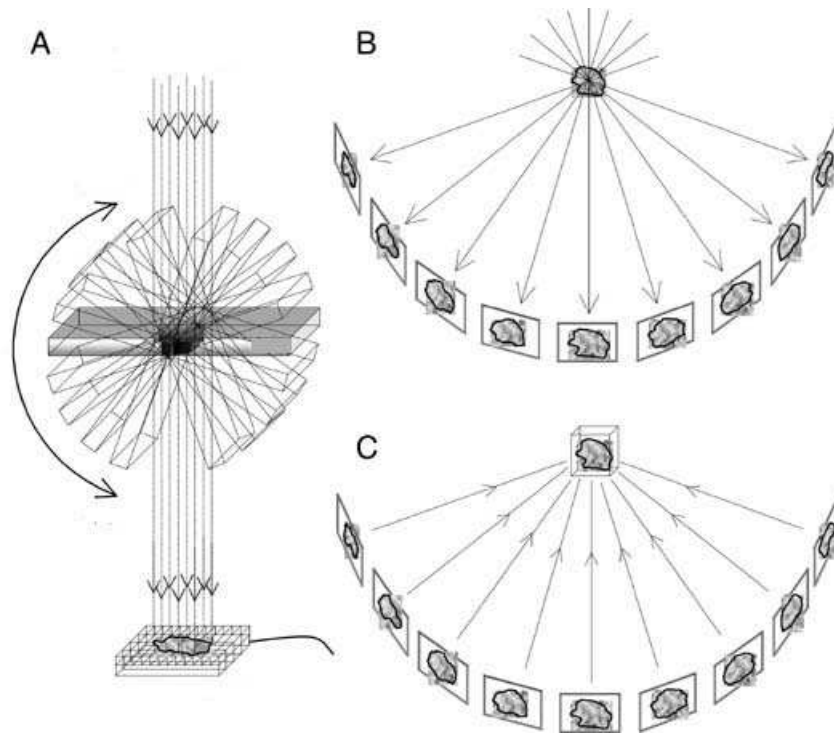


Figure 1: The two steps in Medical Imaging : Projection / Backprojection

The principle is illustrated above : a source of x-rays cross the object and the detector receives attenuation of the beams (i.e a projection). The source often describes a circular trajectory around the object and so produces several projections with different angles (step A,B). Then, some algorithms have to reconstruct the object from those projections (step C, to recover the density function of the object).

2.2 Goal of the intership

Until a couple of years ago, scientists thought that we had to measure all the lines crossing the object to be able to reconstruct it, and consequently to describe all the trajectory around the object. Recently, in 2002, it was proven that it is possible to reconstruct regions of an object from incomplete data, in our case from shorter trajectory, see below :

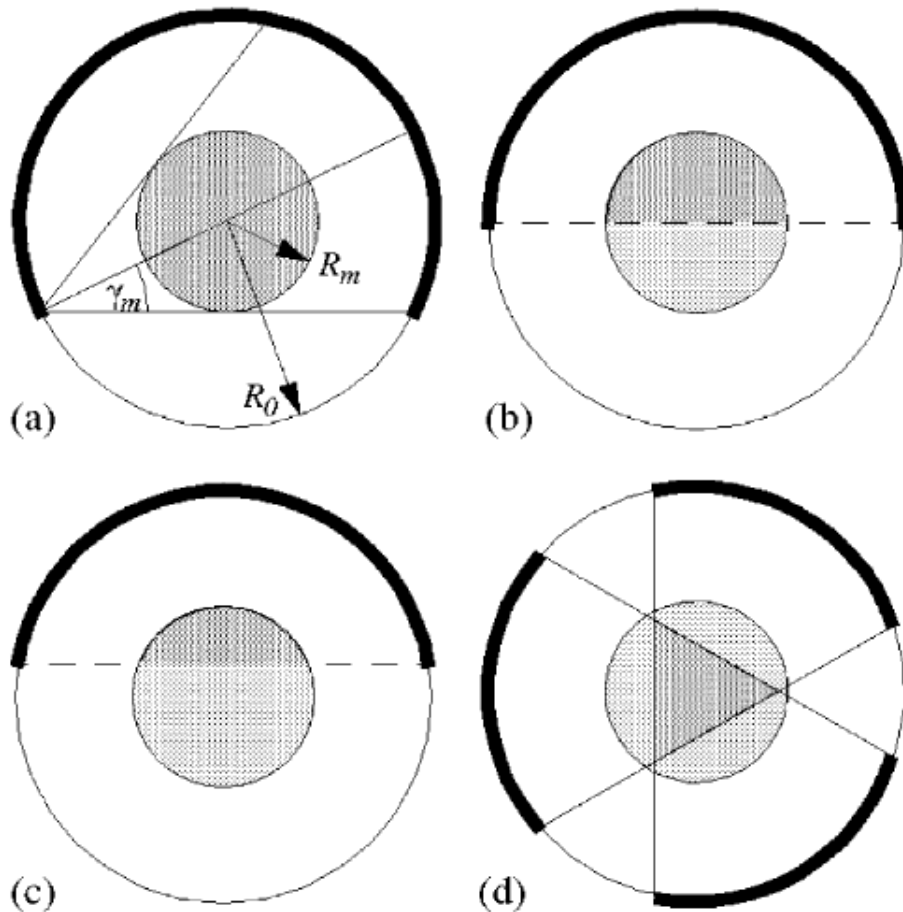


Figure 2: Short-scan and three very short-scans. The object is assumed to occupy the circular shaded region of radius R_m , and the vertex path lies on a concentric circle of radius R_0 . The detector is always large enough to image the full object, thereby accommodating a maximum fan-angle of γ_m . (a) A conventional short-scan of $\pi + 2\gamma_m$ allows reconstruction of the whole object. (b)–(c) A continuous scan of less than $\pi + 2\gamma_m$ allows reconstruction of all object points inside the convex hull of the scan. (d) A scan of three equally spaced segments of 80° each allows reconstruction of a triangular ROI in the centre of the object.

During this internship, I began by understanding this new paradigm and the mathematical concepts underlying it, and then to implement algorithms reconstruction of 2D image with Matlab. Interests at stake are multiple : reduced measurement requirement immediately suggested important dose reductions for some kinds of scans. Also, oversized objects, such as very large patients that exceed the scanner field of view, could be at least partially imaged.

After having exposed some theory elements of mathematical reconstruction, I will present the Matlab program I wrote, with the aim of reconstructing a region of interest of the scan of a brain.

3 Theory

3.1 State of the art

We summarize here the state of the art of 2-D image reconstruction theory at the turn of the century. We first establish some notational conventions. The variables α and β will always represent unit vectors in the plane, whose directions are given by f as follows:

$$\alpha = (\cos \phi, \sin \phi) \tag{1}$$

$$\beta = (-\sin \phi, \cos \phi) \tag{2}$$

The unknown density function will be denoted $f(x) = f(x, y)$, and the projection data will be denoted $p(\phi, s)$, which is the line integral of the density function along the line oriented at angle ϕ from the horizontal (x) axis and at a signed distance s from the origin. So

$$p(\phi, s) = \int_{-\infty}^{\infty} f(r\alpha + s\beta) \text{ for } \phi \in (0, \pi), s \in (-\infty, \infty) \tag{3}$$

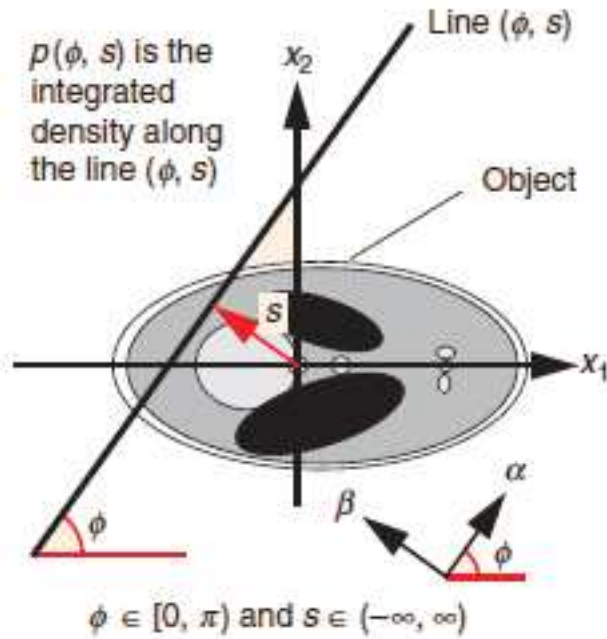


Figure 3: An integral line defined by two variables (ϕ, s) .

The one-dimensional function $p(\phi, \cdot)$ is called the parallel projection of the function f in the direction ϕ . It is rare that a scanner would collect line-integral data in this form of parallel projections, with uniform angular increments and uniform detector increments.

When presented as a 2-D image, the data $p(\phi, s)$ is called the sinogram. Equation (3) expresses the 2-D Radon transform, which maps the density function f to the sinogram p . It is about the first step (the acquisition of all projections).

Then to reconstruct the object, which means recover the density function of the object, we use the Filtered Back Projection (FBP) reconstruction formula :

$$f(x) = \int_0^\pi p_R(\phi, s)|_{s=x \cdot \beta} d\phi \quad (4)$$

with $p_R(\phi, \cdot) = p(\phi, \cdot) * r$ where $r(s)$ is the ideal ramp filter kernel whose Fourier transform is $R(\sigma) = |\sigma|$. Note that $R(\sigma) = |\sigma| = (\frac{1}{2\pi})(2\pi i\sigma)(-i\text{sgn}\sigma)$, so we can express the ramp filter as a derivative composed with a Hilbert transform. Equation (4) can be replaced by

$$p_R(\phi, s) = \frac{1}{2\pi} \frac{\partial}{\partial s} p_H(\phi, s) \quad (5)$$

with $p_H(\phi, \cdot) = p(\phi, \cdot) * h$ where $h(s) = \frac{1}{\pi s}$.

Now a study of the operations on the right hand side from (4) and (5) shows that all values of the sinogram $p(\phi, s)$ are used in the reconstruction formula because the ramp kernel $r(s)$ is known to be nonzero almost everywhere. At each point in the reconstructed image, all nonzero elements of the sinogram make a nonzero contribution to reconstruction. This property of Radon's inversion formula strongly suggests that any missing data will affect the whole image, independent of the algorithm used for reconstruction.

3.2 Solve the problem of incomplete data reconstruction

Fanbeam projections also play a role in partial data problems. From a physical standpoint, fanbeam projections are more natural in the context of X-ray imaging, where the measurement rays all diverge from a point that corresponds to the location of the anode of the X-ray source. We refer to this point as the fanbeam vertex, a . The vertex follows a trajectory around the object, typically a circle outside the scanner port, but we will be more general here and parameterize the movement of the vertex as $a(\lambda)$, with a scalar variable $\lambda \in \Lambda$ where Λ is an interval. We use g to represent fanbeam data :

$$g(\lambda, \phi) = \int_0^\infty f(a(\lambda) + l\alpha) dl \quad (6)$$

(where α is given by (1) as usual). The one-dimensional function $g(\lambda, \cdot)$ is called a fanbeam projection.

We present a mathematical formula with direct consequences for ROI reconstruction from incomplete (yet mathematically sufficient) data. First, the Hilbert transform of fanbeam data is defined to be:

$$g_H(v_\lambda, \phi) = \int_0^{2\pi} g(v_\lambda, \phi') h(\sin(\phi - \phi')) d\phi' \quad (7)$$

and we note that calculation of $g_H(v_\lambda, \phi)$ requires all values of the fanbeam projection $g(v_\lambda, \cdot)$, just as the calculation of $p_H(\phi, s)$ requires all values of the parallel projection $p(\phi, \cdot)$. The parallel-fanbeam Hilbert projection equality is

$$p_H(\phi, s) = g_H(v_\lambda, \phi) \quad \text{where } s = v_\lambda \cdot \beta \quad (8)$$

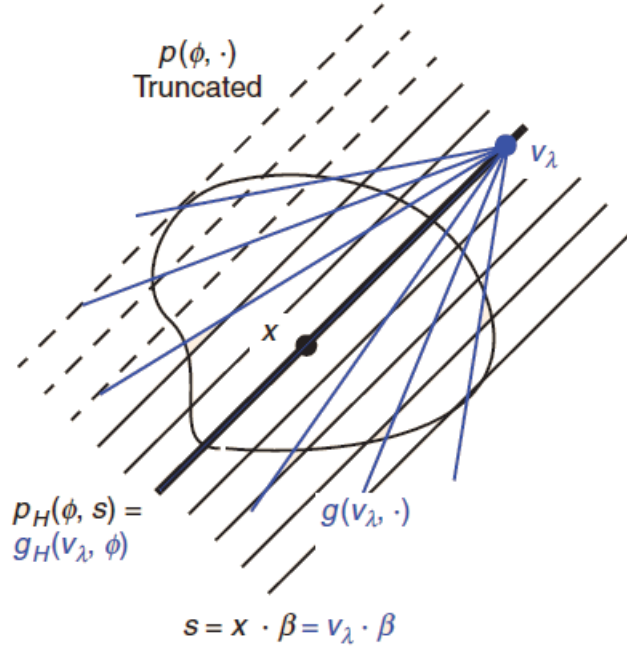


Figure 4: Implications of the Hilbert projection equality. A truncated parallel projection is shown (the dotted lines are unmeasured). According to parallel projection theory, if any projection is truncated, then reconstruction at (any) point x cannot be performed because $p_H(\phi, s)$ cannot be obtained. However, the Hilbert projection equality shows that $p_H(\phi, s)$ might still be obtained via $g_H(v_\lambda, s)$ provided a complete (nontruncated) fanbeam projection $g(v_\lambda, \cdot)$ exists whose vertex lies on the line (ϕ, s) . For data consisting entirely of complete fanbeam projections, the point x can be reconstructed provided a fanbeam vertex lies on each line passing through x .

Equation (8) is of fundamental importance. It shows that there is some flexibility in obtaining $p_H(\phi, \cdot)$, in particular if any values of the $p(\phi, \cdot)$ projection are unavailable (truncated, for example). This is the key point, because it was previously assumed that $p_H(\phi, s)$ could not be obtained for any s if the projection $p(\phi, \cdot)$ was truncated. Instead, we see that $p_H(\phi, s)$ can be evaluated using a fanbeam projection provided (i) the fanbeam vertex v lies on the line (ϕ, s) , and (ii) the fanbeam projection $g(v, \cdot)$ is not truncated.

For the case of a 2-D scan consisting of complete fanbeam projections, the Hilbert projection equality provides a significantly improved data sufficiency condition, allowing partial reconstruction from a fanbeam trajectory on less than a shortscan, that is, from a fanbeam trajectory too short to measure all lines through the object (see Figure 4).

Fanbeam Data Condition: The point x can be reconstructed from complete fanbeam projections provided a fanbeam vertex can be found on each line passing through x (C1)

To sum up, to determine $f(x)$ where x is in the ROI, we have to calculate $p_R(\phi, s)$ or in an equivalent way $p_H(\phi, s)$ in all directions according to (4) and (5) and it is theoretically possible since x verify the condition (C1). But we use a fanbeam source, that's why we apply the Hilbert equality $p_H(\phi, s) = g_H(v_\lambda, \phi)$ and g_H is obtained by (7) where the fanbeam projection $g(v, \cdot)$ is not truncated.

4 Implementation

4.1 Data available

- I worked on an image of a head phantom, called Shepp-Logan phantom, which is a grayscale intensity image that consists of one large ellipse (representing the brain) containing several smaller ellipses (representing features in the brain).



Figure 5: Shepp-Logan phantom (size 128×128)

- The program requires some constants to specify :
 - ★ The image's position, dimension and orientation.
 - ★ The radius R_{traj} of the circular trajectory.
 - ★ The number N_{Vertex} of vertex points on this trajectory.
 - ★ The number N_{beams} of beams emitted at a vertex point.
 - ★ The half-angle maximum γ_m which determine the field of view (FOV).
 - ★ The radius R_{ROI} of the FOV.
 - ★ The discretisation of the circular FOV :

$$\phi_k = k \frac{\pi}{M}, \quad k = 0..M - 1 \quad (9)$$

$$s_l = l \frac{R_{ROI}}{Q}, \quad l = -Q..Q \quad (10)$$

4.2 Architecture

The goal is first to generate fanbeam projections of a ROI of this image in all directions, and then to reconstruct this ROI accurately.

The Matlab program is divided into 3 main files :

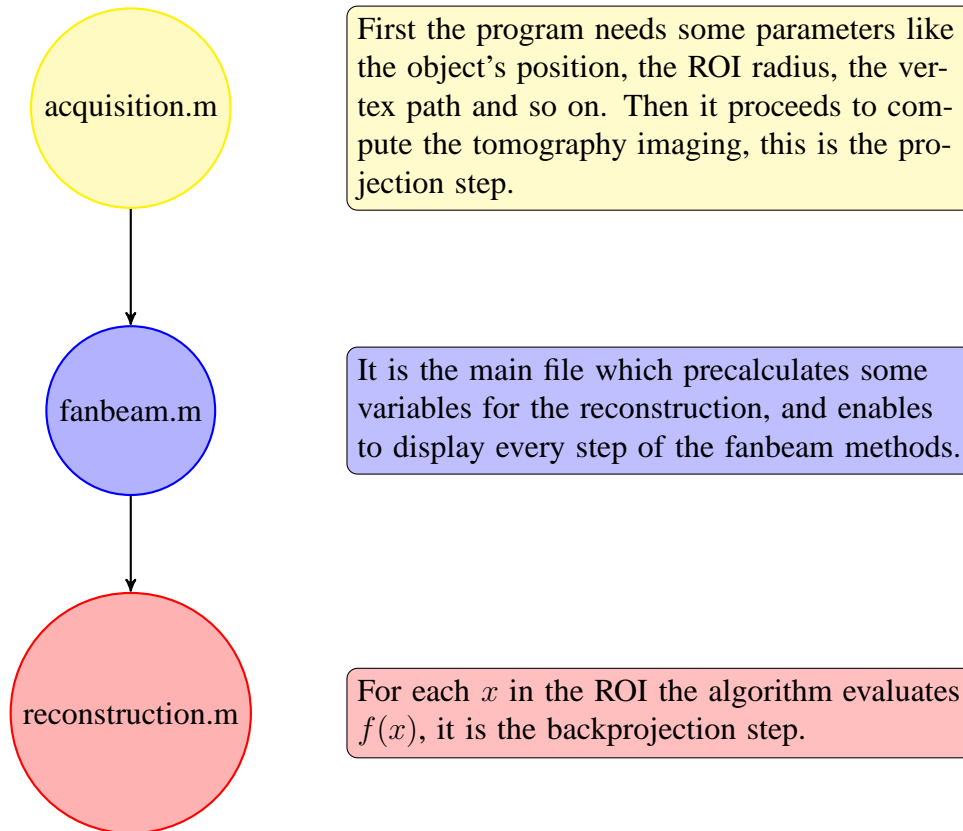


Figure 6: Global architecture of the Matlab program.

4.3 Description of the main functions

4.3.1 acquisition.m

This file calls the following functions :

- `init_data.m` : initializes constants of the previous section and allocates memory for some structures like : the sinogram G , the hilbert filter H , the hilbert transform of the sinogram GH , and ph (resp. php) the array which will contain $p_H(n(\phi_k), s_l)$ (resp. $p'_H(n(\phi_k), s_l)$).
- `fillSinogram.m` : fills the array G (size $NVertex \times Nbeams$) by calculating all $g(\lambda_i, \alpha_j)$, i.e the discrete line integral passing through the image.
- `fillHilbert.m` : fills the array GH (same size that G) resulting of the Hilbert transform applied to G (all $g_H(\lambda_i, n(\alpha_j))$ are determined).

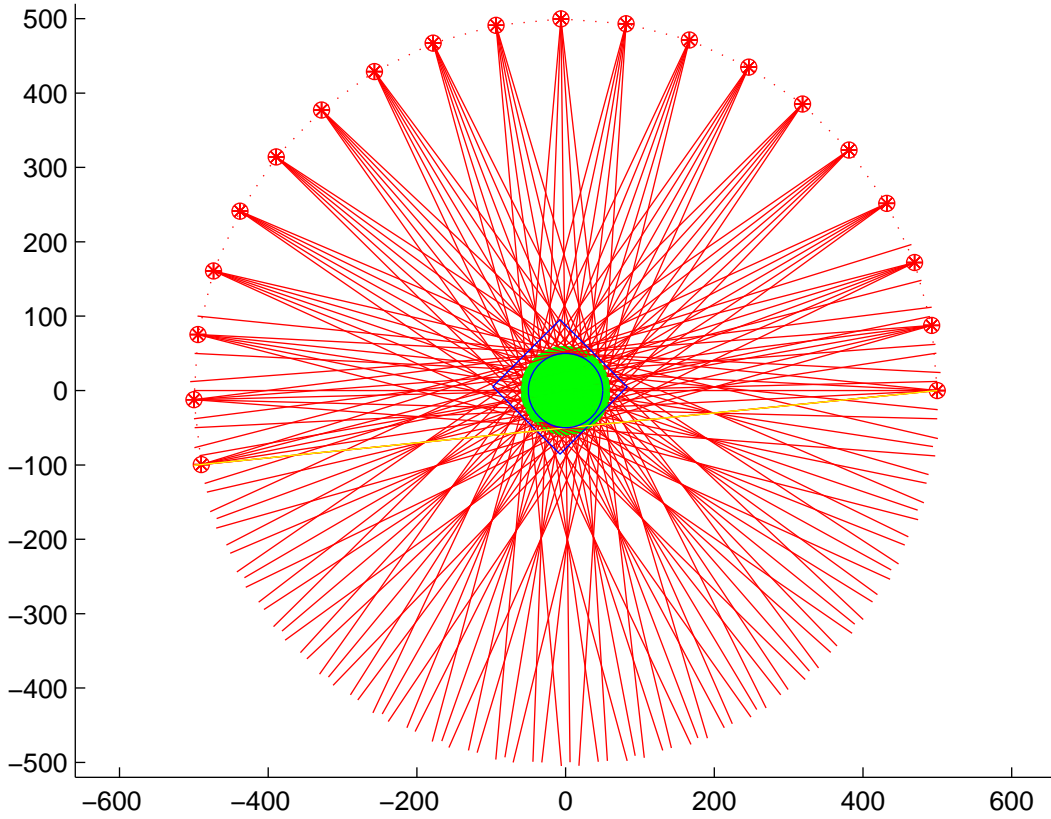


Figure 7: The source of X-rays moves on the vertex path, and for each position (corresponding to a vertex point $a(\lambda_i)$) scans the FOV (blue circle) containing the whole object by calculating $g(\lambda_i, \alpha_j)$ for all α_j . This illustrates the construction of the sinogram G made by the function `fillSinogram.m`

Remark on `fillHilbert`

I had to regularize the hilbert function $h_H(s) = \frac{1}{\pi s}$ because the integral of the convolution g_H (see (7)) was unstable around zero. Recall that the Hilbert transform of a function $g : \mathbb{R} \rightarrow \mathbb{C}$ in the frequency domain is :

$$\widehat{H}g(\nu) = -i \operatorname{sign}(\nu) \hat{g}(\nu)$$

We define H_c the regularized Hilbert transform by :

$$\widehat{H}_c g(\nu) \stackrel{\text{def}}{=} -i \operatorname{sign}(\nu) \chi_{[-c,c]}(\nu) \hat{g}(\nu)$$

We can prove that :

$$H_c g(s) = h_c * g(s) \tag{11}$$

$$h_c(s) = h(s)[1 - \cos(2\pi sc)] = \frac{2}{\pi s} \sin^2(\pi sc) = 2 \operatorname{sinc}^2(\pi sc) c^2 \pi s \tag{12}$$

This new function is now $\mathcal{C}^\infty(\mathbb{R})$.

I chose a cut-off frequency c :

$$c \simeq 200 \frac{Q}{2 \cdot RROI}$$

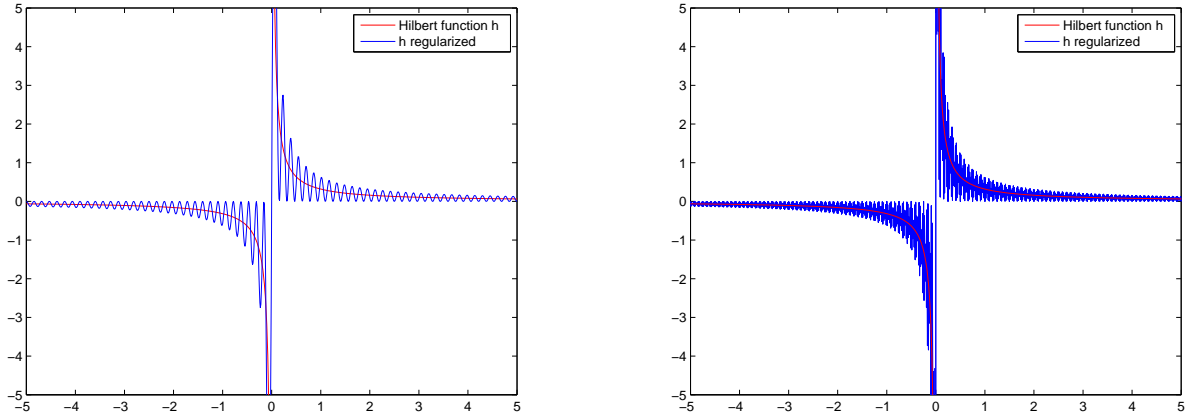


Figure 8: On the left a small cut-off (keep low frequencies), on the right greater cut-off (oscillations frequencies greater). We can see this new function is defined at the origin.

4.3.2 fanbeam.m

I made the effort to produce a Matlab program which could be a teaching aid. Indeed this function enables us to choose a *mode* which correspond to a particular step in the tomography process. Besides, it was a very practical way to debug the program.

- **mode onebeam :**

This mode enables us to display the image boundaries (blue rectangle), the FOV (here equal to the ROI), the vertex path and one beam which cross the image. The red filled point is a vertex point $a(\lambda_i)$ and the red beam correspond to $g(\lambda_i, \alpha_j)$ where α_j is the angle between the black line and the beam. We can also visualize the line integral discretisation in the referential of the image (figure 10). The object we want to reconstruct is inside the FOV, so only green points contribute to the integral.

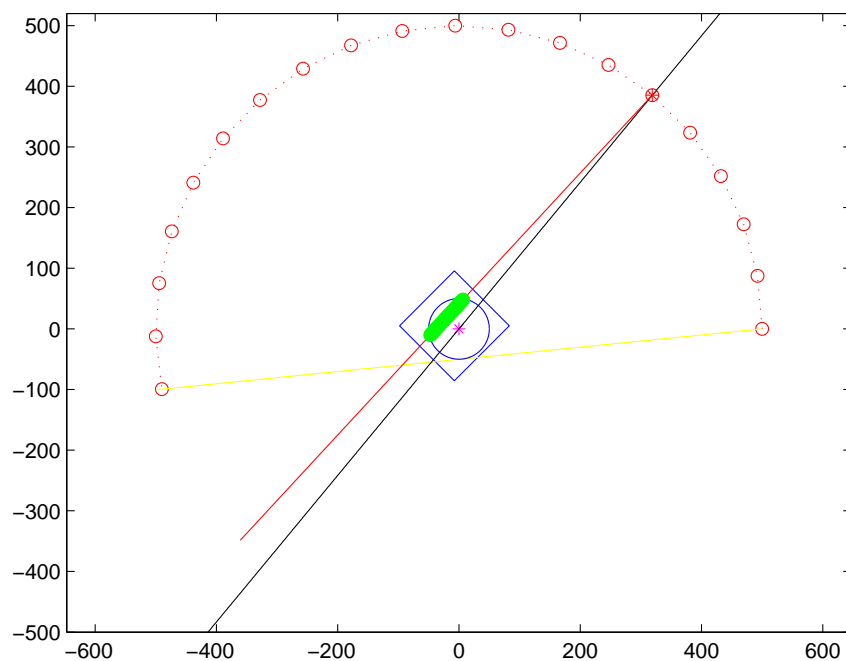


Figure 9: A X-ray crossing the object inside the FOV (here equal to the ROI). The red filled point is a vertex point $a(\lambda_i)$ and the red beam correspond to $g(\lambda_i, \alpha_j)$ where α_j is the angle between the black line and the beam.

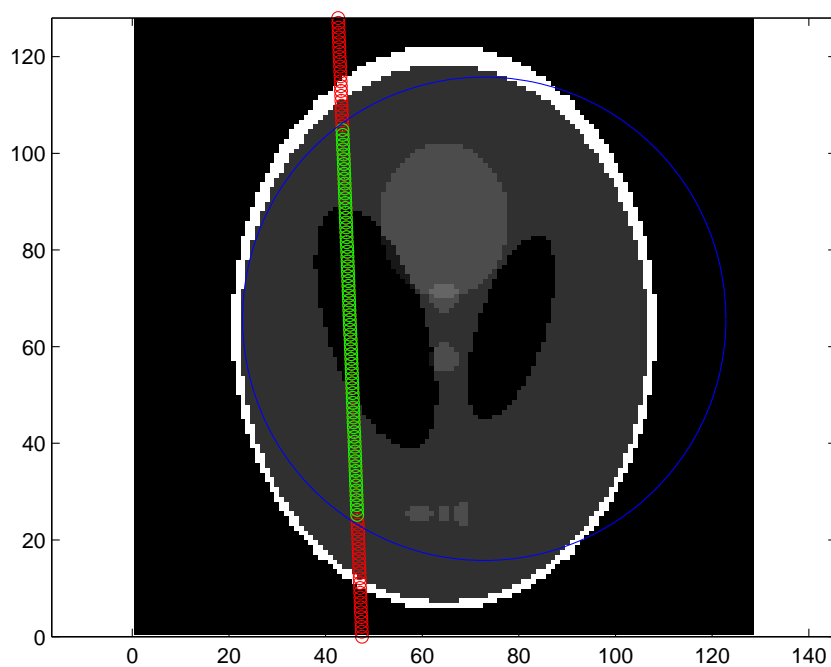


Figure 10: The discrete line integral seen in the image referential. Only green points (in the FOV) contribute to the projection calculation.

• **mode rebinning :**

In this mode we can play with the Hilbert equality by considering the illustration of rebinning, i.e the calculation of p_H . Let us see how it works to evaluate $ph(k, l) = p_H(n(\phi_k), s_l)$:

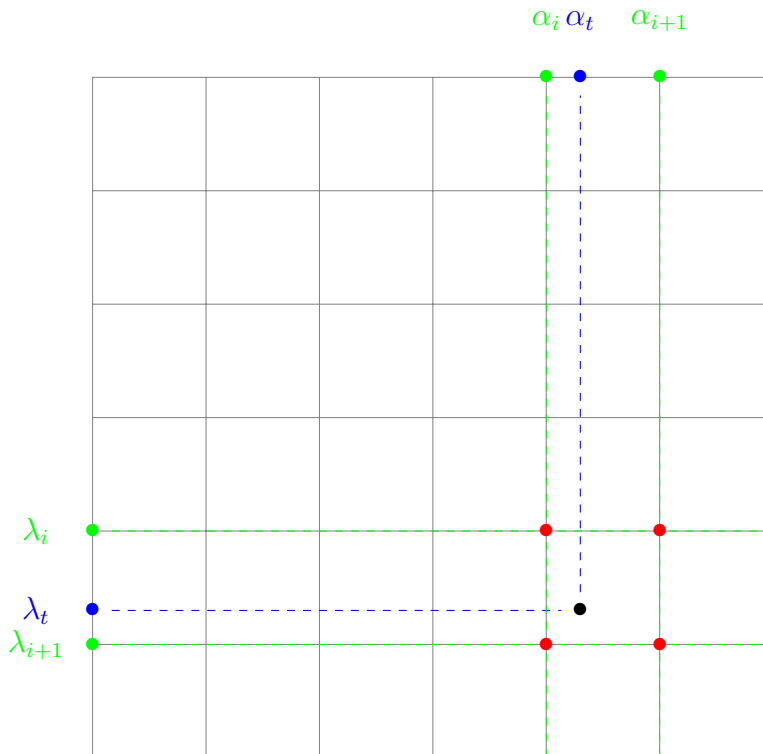
1. First we look for the intersections of the line $(n(\phi_k), s_l)$ (black line on figure 12) with the circle (radius R_{traj}) what gives two points $P1 = a(\lambda_1)$ and $P2 = a(\lambda_2)$, outputs of the function `intersec_path.m`. The variables `P1in` and `P2in` precise if these points belong to the vertex path. If it is the case the point appears on blue on the graph (else on yellow).

2. Then we determine the angle $\alpha_t = (P_t, \vec{P_t O}, P_t \vec{P_{kl}})$ where $P_{kl} = s_l n(\phi_k)$ (black point), $P_t = a(\lambda_t)$ the point previously found and O the origin (magenta point). This is the role of the function `alphanat.m`.

3. We also determine the indices i and j (resp. by the function `findVertex.m` and `findAngle.m`) such that $\lambda_i \leq \lambda_t \leq \lambda_{i+1}$ (the two points $a(\lambda_i)$ and $a(\lambda_{i+1})$ are in green on the figure) and $\alpha_j \leq \alpha \leq \alpha_{j+1}$.

4. Theoretically the Hilbert equality stipulates that $p_H(n(\phi_k), s_l) = g_H(\lambda_t, \alpha_t)$ but as we work with a discretisation we have to carry out a bilinear interpolation (made by the function `interpolBilGH.m`)

$$g_H(\lambda_t, \alpha_t) \simeq g_H(\lambda_i, \alpha_j)(\lambda_{i+1} - \lambda_t)(\alpha_{j+1} - \alpha_t) + g_H(\lambda_{i+1}, \alpha_j)(\lambda_t - \lambda_i)(\alpha_{j+1} - \alpha_t) + g_H(\lambda_i, \alpha_{j+1})(\lambda_{i+1} - \lambda_t)(\alpha_t - \alpha_j) + g_H(\lambda_{i+1}, \alpha_{j+1})(\lambda_t - \lambda_i)(\alpha_t - \alpha_j)$$



GH

Figure 11: Bilinear interpolation of $g_H(\lambda_t, \alpha_t)$

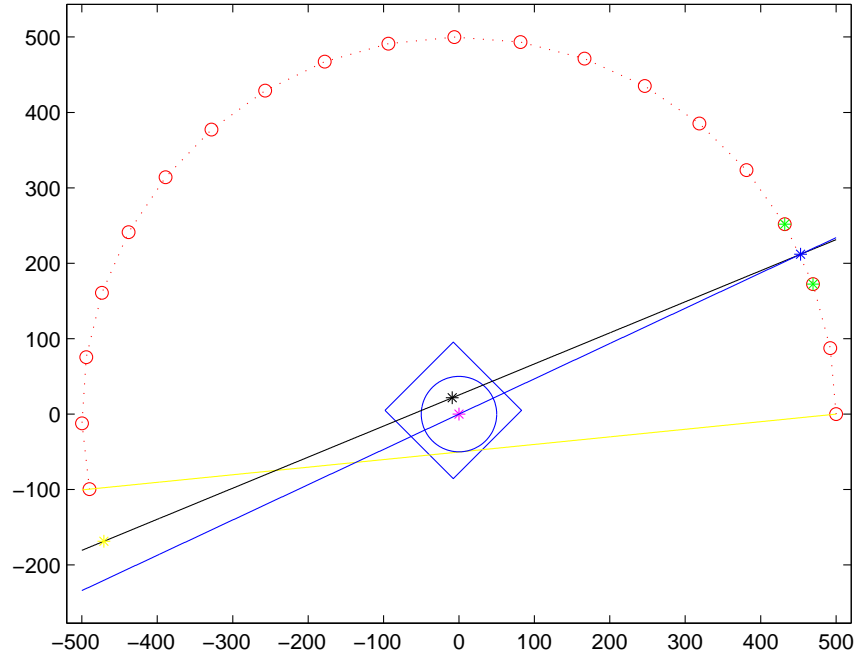


Figure 12: 1. The intersections between the line $(n(\phi_k), s_l)$ and the circle are determined (blue and yellow points) ; 2. The angle α_t between the black and blue line is determined ; 3. We search in arrays *lambda* and *angles* the values such that $\lambda_t \in [\lambda_i, \lambda_{i+1}]$ and $\alpha_t \in [\alpha_j, \alpha_{j+1}]$

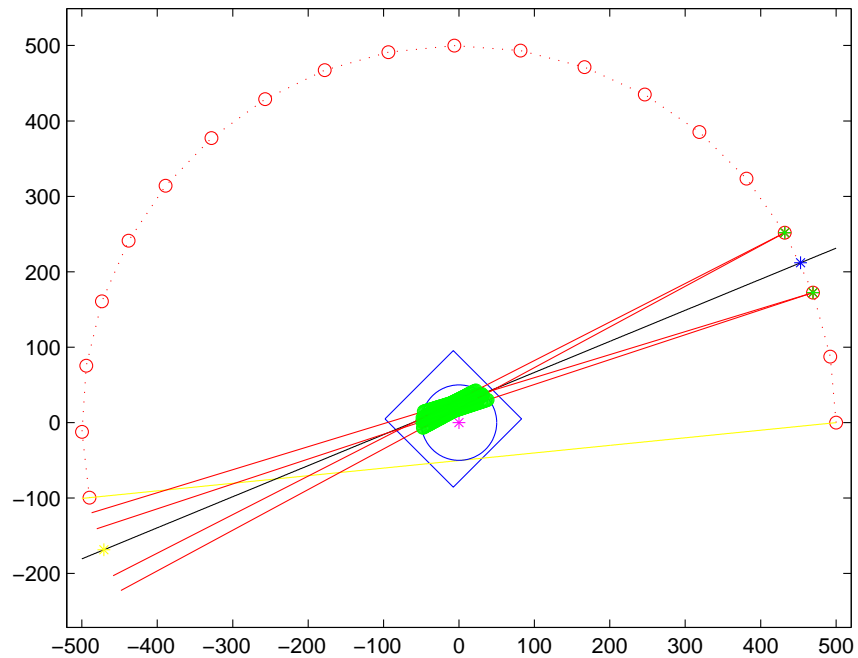


Figure 13: 4. The four beams $g_H(\lambda_i, \alpha_j)$, $g_H(\lambda_{i+1}, \alpha_j)$, $g_H(\lambda_i, \alpha_{j+1})$ and $g_H(\lambda_{i+1}, \alpha_{j+1})$ are plotted.

The previous part explains how the function `calculate_ph.m` proceeds, this is the way it fills the

array $ph(k, l)$. The function `calculate_php` can from now fills the php array :

$$p'_H(n(\phi_k), s_l) \simeq \frac{p_H(n(\phi_k), s_{l+1}) - p_H(n(\phi_k), s_{l-1}))}{s_{l+1} - s_{l-1}} \quad (13)$$

At the end of this second part, we realized this serie of operations :

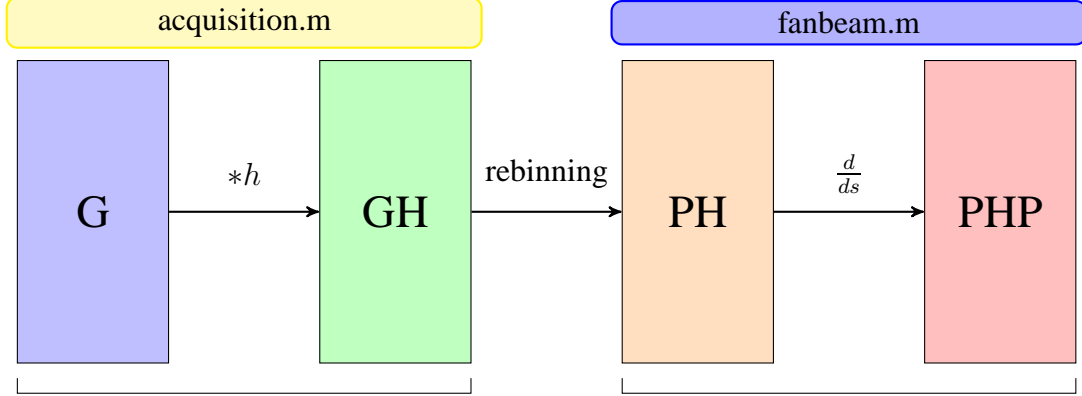


Figure 14: Illustration of the operations made in chain on the arrays before the reconstruction step.

4.3.3 reconstruction.m

Finally $f(x)$ is computed according to the formula :

$$f(x) \simeq \frac{1}{2M} \sum_{k=1}^M p'_H(n(\phi_k), x \cdot n(\phi_k)) \quad (14)$$

using linear interpolation between the two samples of s nearest to $x \cdot n(\phi_k)$.

If $x \in ROI$ then it will reconstruct accurately.

5 Results and analysis

These results are obtained with the following parameters :

- * $R_{traj} = 500$
- * $RROI = 50$
- * $RROI = 50$
- * $NVertex = 256$
- * $Nbeams = 512$
- * $\gamma_m = \arcsin(\frac{RROI}{R_{traj}}) \simeq 6^\circ$
- * $Q = 128, M = 401$.

The source is far enough from the object ($R_{traj} \gg RROI$) to imagine that the sinogram will look like a sinogram in the parallel geometry since the beams will be almost parallels due to the distance. Indeed we obtain a pretty good result :

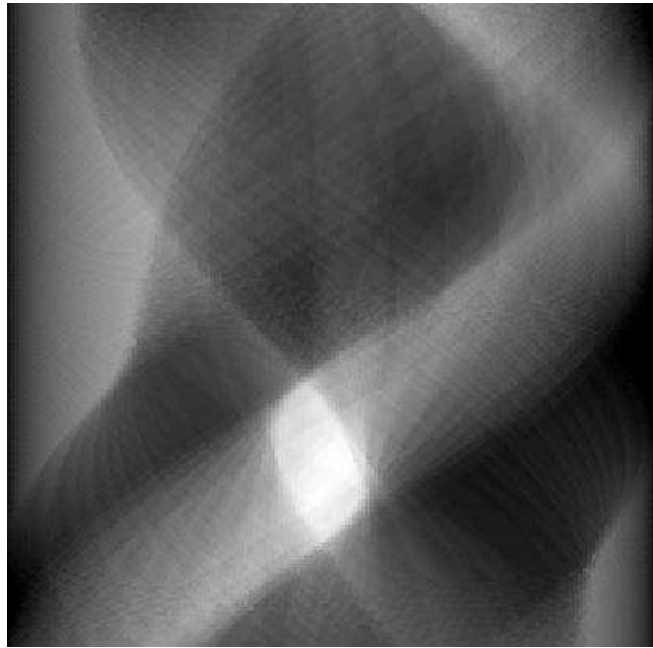


Figure 15: Sinogram of a part of the Shepp-Logan phantom in a fanbeam geometry almost equivalent to a parallel geometry.

This sinogram G has the usual form. Applying the regularized Hilbert filter on this sinogram, we get for GH an image close to the sinogram display but smooth, which seems logical.



Figure 16: Sinogram filtered by the Hilbert filter.

The rebinning enables getting from fanbeam geometry to parallel geometry, producing this image for PH :

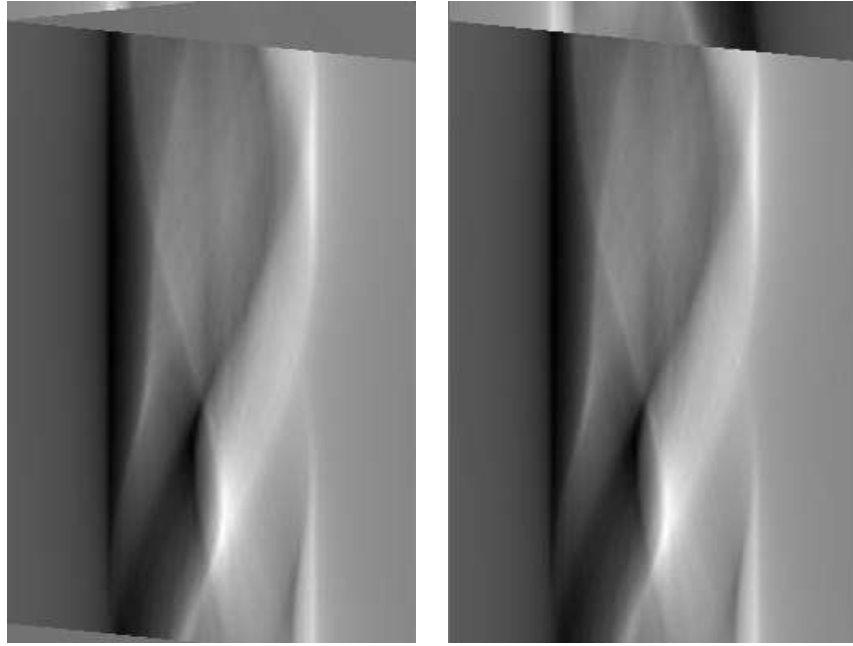


Figure 17: Image corresponding to PH after rebinning. Two regions present a sign problem, that is half fixed by taking the difference instead the mean.

The image seems to be good, but as you can notice, there is a problem in uniform grey region (all pixels values are 0). I understood these regions as corresponding to line $(n(\phi_k), s_l)$ which intersect the vertex path in two points $a(\lambda_1)$ and $a(\lambda_2)$. In my initial program I calculate the mean of each contribution $g_H(\lambda_1, \alpha)$ and $g_H(\lambda_2, -\alpha)$, but it turns out that $g_H(\lambda_1, \alpha) \simeq -g_H(\lambda_2, \alpha)$. That's why I rather take the difference, it solves the problem just for one region, the other has an opposite grey level (image on the right). I will try to fix that point before the end of the intership. In any case this inversion of contrast isn't really a problem because the next step calculate the derivative, so only variation is interesting.

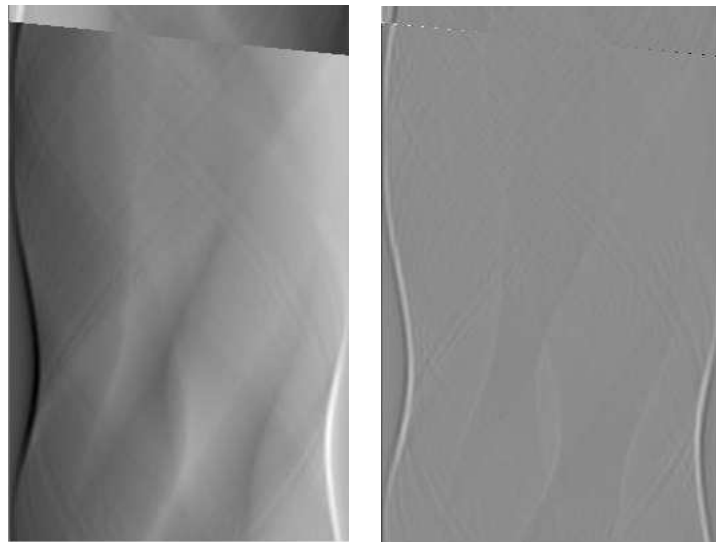


Figure 18: Image corresponding to PH after rebinning.

The problem is just visible on a line, what means only one index k_0 will give a bad value for $p'_H(n(\phi_{k_0}), x \cdot n(\phi_{k_0}))$ in the sum of M terms. The effect is negligible among the $M = 401$ values taken. Indeed we finally obtain a good reconstruction :

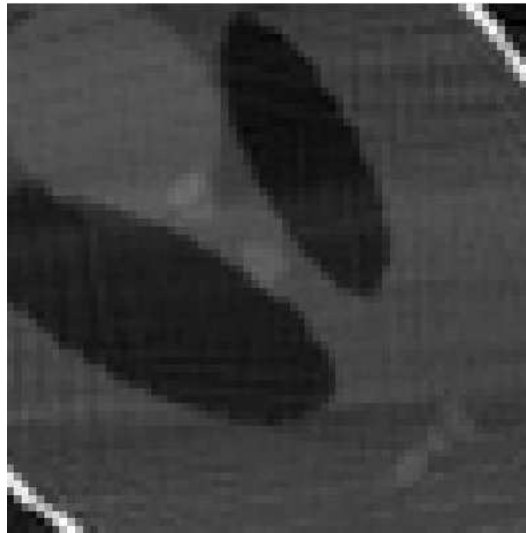


Figure 19: Image reconstructed (the square inscribed in the FOV circle)

6 Personal record

I enjoyed discovering a new sector in the field of informatics and mathematics applied to the healthcare (I already made an initiation to the research in laboratory on the theme of neurosciences). I was introduced to a new area of maths, and above all I realized once again how difficult it is to get a robust implementation from a theoretical analysis (in a short period of two months). Indeed many times I grappled with numerical problems which don't exist in theory. For instance some comparisons between numbers (like $45 \geq 45.000$) appear false, divisions by zero, singular matrix, or instability problems (as the problem of the convergence of the integral g_H that I was compelled to regularized) and so on. This kind of error is often difficult to identify, so I aimed to divide my program into many functions that could be tested individually. Moreover, the different *modes* I set up were very useful in identifying where the program failed by visualizing the process on plot.

Besides I discovered the business world and all the problems it implies : the economic dimension, we have to find funds before we can launch a project; the competition between companies and the question of confidentiality : patents must be taken seriously. I didn't realize all of that during my university studies, but what I appreciated in company is to work within a team whose members come from a variety of backgrounds. That point was very interesting, and I liked being plunged in the mix of mathematics, informatics, robotics and medicine.

7 Conclusion

At that time, all the processes of reconstruction in two dimensions work in Matlab for a short scan trajectory. The next step will be first to fix the related error, and to check that the reconstruction still works for less than a short scan trajectory. Then I will bring some improvements to accelerate the execution of the program, and eventually I would like to quantify my results (noise resistance, stability, and so on).

In the future we can imagine that this algorithm will be adapted in C++ (to gain more performance) and integrated in 3D reconstruction.

8 Bibliography

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